A More Accurate Approach for calculating Illuminance with Daylight Coefficients
Sarith Subramaniam$^1$, Richard G. Mistrick$^1$

1. Department of Architectural Engineering, Pennsylvania State University, USA

Daylight coefficients form the basis for most conventional daylight simulation software. In many cases, this involves simplifying assumptions pertaining to the discretization of the sky into patches as well as locations of the solar disc in the celestial hemisphere. This study sought an improved model by examining the benefits of utilizing accurate sun positions as well as appropriate levels of sky discretization.

The performance of a currently applied Standard Daylight Coefficient Model was compared to that of the improved model proposed by the authors for a south-facing space. Radiance-based simulations were performed for ten hours during solstice and equinox dates for weather data pertaining to six U.S. cities with varying climates. The results from both models were compared with benchmark results obtained through conventional ray-tracing. The illuminances obtained from the standard model were found to deviate from the benchmark simulation by over 40% for certain hours with direct solar insolation. For the same instances, the deviations observed in the illuminances calculated through the improved approach proposed by the authors deviated from the benchmark by approximately 5%, thereby demonstrating an improvement in accuracy. From a location-based perspective, the deviation in Annual Sunlight Exposure between the standard model and the proposed model was found to be highest for Phoenix-AZ at a value of 2%. The improvements in the results from the proposed model can be attributed to the use of accurate sun positions alone. A comparison of data from simulations involving different levels of sky discretization did not indicate any impact on accuracy. Conversely, the simulation runtime for daylight coefficients was found to directly relate to the level of sky discretization.

Sarith Subramaniam is a Ph.D. candidate in Architectural Engineering from Penn State University. He holds a MS in Architectural Engineering, with focus on Lighting, from Penn State University.

Richard Mistrick, PhD, is an Associate Professor of Architectural Engineering at Penn State, where he teaches and conducts research in illumination engineering, including the areas of lighting and daylight systems modeling, design and performance evaluation, as well as daylight integrated lighting controls.

1. Introduction and Background
Annual daylighting simulations inform the design decisions pertaining to several aspects of a building. Factors such as building orientation, window-to-wall ratios, glazing material selection and the incorporation of shading elements into the building mass are influenced by the results obtained through such simulations. The importance of annual daylighting simulations, and the metrics generated through them, has also been acknowledged in building design codes and standards (DiLaura and others 2011; IES 2012; USGBC 2014). As is the case with any computer-based simulation meant to predict or mimic real-world phenomena, the quality of results obtained through daylighting simulations depends on the input parameters as well as the mathematical model employed for the calculations. The input parameters refer to the geometry and surface properties of the space being studied. The geographical location and typical meteorological radiation data of the site are also part of the input, and are employed to create a physically-based definition of the sky and sun. The mathematical model relates to the equations and algorithms that utilize the input data to accurately predict the daylighting conditions expected to prevail inside the space.
For more than two decades, the Radiance ray-tracing software has been acknowledged and validated as a reliable calculation engine for daylighting simulations (Ochoa and others 2012). Provided the inputs are accurately measured and specified, Radiance and Radiance-based software have been able to predict point-in-time illuminance values within a margin of less than ±10% error (Mardaljevic 1999; Mardaljevic 2001). An annual daylighting simulation typically involves calculating illuminance values for more up more than 3000 hours a year. Repeating a point-in-time simulation for each of those hours is prohibitively expensive. Consequently, researchers have endeavored to devise computationally efficient methods for performing annual daylighting simulations by employing certain simplifying assumptions. The most prominent among these methods is the Daylight Coefficient Method (Tregenza and Waters 1983). The Daylight Coefficient approach was first validated with Radiance by Mardaljevic (1999) and subsequently implemented in the Daysim software (Reinhart and Walkenhorst 2001). Daysim is a derivative of Radiance and is the calculation engine for several GUI-based tools such as DIVA4Rhino, Ladybug-Honeybee and SPOT (Daylighting Innovations LLC 2011; Jakubiec and Reinhart 2011; Roudsari and Pak 2014).

One of the basic assumptions of the Daylight Coefficient Method is that the illuminance inside a space can be calculated by summing up the luminous contributions from individual patches of a discretized hemispherical sky. This concept is illustrated schematically in Figure 1. Tregenza’s initial sky model, depicted in Figure 2, consisted of 151 circular patches.

![Figure 1. A schematic description of the Daylight Coefficient Method. The term DC in the equation denotes Daylight Coefficient. (Reinhart 2001)](image)

The size of the patches was chosen per the angular probe-size of a popular sky scanning photometer employed at the time (Tregenza 1987). In subsequent years, the availability of weather tapes through Typical Meteorological Year (TMY) data, and the development of the Perez Sky model led to the use of a mathematically generated continuous sky model in daylighting simulations (Perez and others 1991; Perez and others 1993). Mardaljevic (1999), and subsequently Reinhart and Herkel (2000), proposed a sky model with 145 trapezoidal-like patches on the sky dome for use with Perez Skies. Present day sky models used in Daylight Coefficient simulations are based on Perez Skies with these, or further discretized, patches. The other simplifying assumption in the Daylight Coefficient Method relates to the position of the sun and the number
of solar discs considered in the simulation. The original implementation of Daysim approximated the location of the sun to around 65 locations in the sky for the entire year as shown in Figure 3. The Standard Daylight Coefficient Model improved upon this assumption by proposing a model with 2305 locations (Bourgeois and others 2008). The position of the sun in this scheme is based on the center of a Reinhart sky-subdivision where each of the 145 patches is further divided into 16 individual patches, thus resulting in 2305 locations for the solar disc \(144 \times 16 + 1\). A comparison between the actual locations of the solar disc and those designated through Reinhart-subdivisions is shown in Figure 5.

Figure 3. In the initial version of Daysim, the position of the sun at a given hour in year was assumed to be one among the 65 locations like those highlighted by crosses in the above image. The faint dots in the background denote the actual position of the sun on an hourly basis throughout the entire year. (Reinhart and Walkenhorst 2001).
Figure 4. A comparison between Tregenza sky subdivision scheme and the continuous scheme proposed by Mardaljevic (1999) and adopted by Reinhart and Walkenhorst (2001) in Daysim.

Figure 5. In the above figures, the actual hourly location of the solar disc is compared with that generated through progressively higher configurations of Renihart subdivision scheme. The number of suns generated through a particular “MF” option can be calculated as (MF × MF × 144 + 1).
This research investigates the impact of employing precise sun positions and highly discretized skies on the accuracy and runtime of annual daylighting simulations. The authors hypothesize that illuminance results thus obtained will be closer to values obtained through conventional point-in-time raytracing calculations, and therefore more accurate. The following section describes the proposed model in detail.

2. Theoretical basis

The discretized sky used in the Standard Daylight Coefficient method is meant to approximate a continuous sky defined through the Perez all-weather sky model. As shown in Figure 6, employing higher levels of discretization for the sky will lead to a closer approximation of the continuous sky. This is especially pertinent in the case of clear and intermediate skies where there is a sharp change in the luminous gradient in the circumsolar region. Traditionally, the use of a higher number of sky patches has been associated with a proportional increase in simulation runtimes for Radiance-based calculation methods. The development of the rcontrib tool in Radiance, named formerly as rtcontrib, has made it possible to perform such calculations within a much shorter time. Unlike traditional Radiance tools such as rtrace and rpict, which rely on ambient caching algorithms, rcontrib relies on pure Monte-Carlo methods for raytracing (Ward 2012).

![Figure 6](image.png)

Figure 6. The image on the left shows a falsecolor fisheye representation of a sky that relates to the CIE definition of a Standard Clear Sky with low luminous turbidity (CIE 2014). Continuous sky definitions like the one in image (a) are used in conventional ray-tracing simulations. The subsequent images (from left to right) show discretized sky definitions that are approximations of the continuous model. The discretized skies are generated using Reinhart sky subdivisions. Images (b), (c) and (d) contain 145, 577 and 1297 patches respectively. The sky definition with 145 patches is most commonly employed in Daylight Coefficient simulations including the Standard Daylight Coefficient Model.

For the same model and input parameters, Monte-Carlo methods are computationally more economical for simulations where the primary aim is to calculate the contribution of known sources of light to a scene (Jacobs 2010).

The second aspect of the Standard Daylight Coefficient Model addressed by this study focuses on the positions and quantity of solar discs considered in the simulation. As per the original implementation of daylight coefficients through Daysim, the number of suns was restricted to 65. As discussed previously, the Standard Daylight Coefficient Model increased this value to 2305. The authors propose the use of precise sun locations by including a solar disc for each hour during which the sun is above the horizon and not occluded by clouds. While the facility to generate solar discs in such a manner is not currently available in a native Radiance program, the same functionality has been provided in other software that use Radiance as a rendering engine (Casey and Mistrick 2015; Roudsari and Subramaniam 2016). When plotted for the entire year on an hourly basis, the solar discs together manifest as analemma patterns as shown previously in Figure 5. Each analemma pattern indicates a particular hour of the day that is simulated across all 365 days of the year.
The impact of using highly discretized skies and precise sun locations on daylight coefficient simulations was evaluated by comparing the performance of a new model, one that incorporates these features, with the Standard Daylight Coefficient Model. The following section details the methodology employed for this study.

3. Methodology

All daylighting simulations performed in this study employed a simple south facing room shown in Figure 6. This room was lit through sidelighting by a single aperture. The dimensions of the room are detailed in Figure 7 and surface reflectances and glazing transmittance are listed in Table 1. The illuminance measurements were performed for 100 sensors at a height of 2.5 feet from the floor. All simulations were performed for six different locations in the US. The locations were chosen based on similar latitudes but differing sunshine conditions. These locations are depicted in Figure 9. The illuminance study was

![Figure 7. The space used for the simulations is a south facing room with a floor dimensions of 20’x20’ and a height of 10’ from ceiling to floor. The south orientation was chosen for simulations because it is highly susceptible to direct insolation.](image)

Table 1. The optical properties of different surfaces considered in the simulations. All opaque surfaces were assumed to be Lambertian.

<table>
<thead>
<tr>
<th>Surface</th>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ceiling</td>
<td>Reflectance</td>
<td>90%</td>
</tr>
<tr>
<td>Walls</td>
<td>Reflectance</td>
<td>65%</td>
</tr>
<tr>
<td>Floor</td>
<td>Reflectance</td>
<td>30%</td>
</tr>
<tr>
<td>Ground</td>
<td>Reflectance</td>
<td>20%</td>
</tr>
<tr>
<td>Glazing</td>
<td>Transmittance</td>
<td>65%</td>
</tr>
</tbody>
</table>
Figure 8. Location of calculation points considered for simulations. As the numbers in the figure indicate, each calculation point is spaced 2’ in x and y directions from the other point. All the points are 2’6” inches from the floor level.

Figure 9. Location of the six cities considered for simulations. The paired locations were chosen according to similar latitudes but differing climatic conditions (Ai 2016).

performed for 10 hours from 8:30 to 17:30 on the solstice (June 21st and December 21st) and equinox dates (March 20th and September 22nd).

The performance of the Standard Daylight Coefficient Model and that of the improved model proposed by the authors were compared against benchmark results generated through conventional ray-tracing. The calculation parameters for each ray-tracing simulation were iteratively improved till the successive results from two sets of parameters converged. Convergence in this context was defined as the condition that the illuminance values at every individual calculation point between two iterations is within a margin of ±2% difference. Of the 240 instances of benchmark simulations performed for six locations and forty hours,
convergence for all but eight instances occurred within the calculation parameters of (-ab 6, -ad 2048, -ar 3072 and -aa 0.05). For the eight outlying instances, convergence occurred when the ambient accuracy was raised to 0.01, while the other parameter values were retained.

Table 2. Simulation parameters considered for simulating conventional ray-traced models till convergence of results.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Options</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ambient Bounces (-ab)</td>
<td>5, 6, 7, 8</td>
</tr>
<tr>
<td>Ambient Divisions (-ad)</td>
<td>1024, 2048, 3072, 4096</td>
</tr>
<tr>
<td>Ambient Resolution (-ar)</td>
<td>1024, 2048, 3072, 4096</td>
</tr>
<tr>
<td>Ambient Accuracy (-aa)</td>
<td>0.5, 0.1, 0.05, 0.01</td>
</tr>
</tbody>
</table>

The first stage of both methods involved calculating illuminance by tracing rays to a sky consisting of discretized patches that accounted for the radiation from the sun and the sky. The second stage in both methods involved the subtraction of the direct illuminance contribution from the sky patches that pertains to the sun within the first simulation. Finally, the direct contribution from the sun was calculated and added into the results from the second stage. This approach utilizes the Standard Daylight Coefficient Method to compute the reflected contribution from sunlight, but addresses the direct contribution with a more accurate modeling method.

Table 3. Simulation parameters considered for Daylight Coefficient Simulations. The value of limit weight, specified through the “-lw” flag in Radiance parameters, was calculated as the reciprocal of the amount of ambient divisions multiplied by 0.01 to allow the contribution of each of the rays traced in the simulation to be accounted for in the final result (Ward 2012).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Options</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ambient Divisions</td>
<td>10E4, 20E4, 30E4, 40E4, 50E4, 60E4, 70E4, 80E4</td>
</tr>
<tr>
<td>Limit Weight</td>
<td>1/(Value of -ad * 0.01)</td>
</tr>
<tr>
<td>Ambient Bounces</td>
<td>5, 6, 7, 8</td>
</tr>
</tbody>
</table>

4. Results

Studies involving validation or comparison of daylighting simulations have traditionally relied on comparing the results between fewer than 10 illuminance measurement sensors at a given time (Mardaljevic 1995; McNeil and Lee 2013; Reinhart and Breton 2009; Reinhart and Walkenhorst 2001). A few recent studies that considered a higher quantity of illuminance sensors have advocated the use of Total Annual Illuminance, which involves the summation of illuminance for every instance of measurement at a particular sensor location (Brembilla and others 2015a; Brembilla and others 2015b). As the current study involved 100 illuminance sensors and was meant to identify the variations in illuminance results as a function of calculation method, none of the previous approaches could be reliably employed for this purpose. Instead, the authors employed comparisons between percentile ranges of illuminance to identify the variation in simulation results between the different calculation methods. Figure 10 and Figure 11 visualize two such comparisons for point-in-time simulations for Denver, CO and Phoenix, AZ. Deviations of up to ±50% between illuminance values calculated through raytracing and the Standard Daylight Coefficient Model were observed for several instances in which direct sunlight was incident on the grid points considered for illuminance measurement from only one of the two calculation methods. These deviations were primarily restricted to values in the 76th to 100th percentile of the measured illuminances. In those instances, the deviations in the illuminance values generated through the approach proposed by the authors were found to
Figure 10. Illuminance values calculated through the different calculation methods. The left-most plot on the top row visualizes the benchmark result obtained through ray-tracing. The plot on its right corresponds to the Standard Daylight Coefficient Model (also known as Dynamic Daylighting Simulation, hence the abbreviation of “DDS”). Subsequent plots correspond to simulations performed with the new method proposed by the authors. The plot with the text “MF1” on top refers to a simulation involving precise sun positions and a sky with 145 patches. Similarly, “MF3” corresponds to a simulation with precise sun positions and a sky with 1297 patches. The letters L, M, and H refer to the mean (and corresponding standard deviation) illuminance values for the 0-25th Percentile, 26th-75th Percentile and 76th-100th Percentile respectively. In this instance, it is apparent that the deviation in results highlighted in the case of the “DDS” simulation can be attributed to the variation in the “H” (high) range.

Figure 11. Like Figure 10, the above image also shows a comparison between illuminance values calculated through the different simulation methods. In this instance, the deviation in the results generated through the Standard Daylight Coefficient model is negative as the mean illuminance in the 76th-100th percentile (“H”) range has been underestimated.
Figure 12. Deviation between daylight coefficient-based methods in mean illuminance results for Birmingham, AL. The results are categorized by percentile ranges as indicated by the title above each sub-plot. In the legend, DDS corresponds to the Standard Daylight Coefficient Model while MF1, MF2…MF6 correspond to the improved method proposed by the authors. As previously explained MF1, MF2 ..MF6 allude to the number of sky-patches present in the discretized sky used of the daylight coefficient calculation.

Figure 13. Mean deviation results for Denver, CO.
Figure 14. Mean deviation results for Fargo, ND.

Figure 15. Mean deviation results for Phoenix, AZ.
Figure 16. Mean deviation results for Pittsburgh, PA.

Figure 17. Mean deviation results for Seattle, WA. The two major spikes in the above image are due to extremely low illuminance values on the work plane that were observed at 13:30 during summer solstice (21st June) and 15:30 on Autumn Equinox (September 22nd). The illuminance plots for these dates are provided in Figure 18 and Figure 19.
be less than ±10%. The deviation in percentile illuminance values calculated from the hourly simulations for the six locations for solstice and equinox dates are summarized in Figure 12 to Figure 17.

5. Discussion
The plots in Figure 12 to Figure 17 indicate that, when compared with the illuminance obtained through the benchmark raytrace simulations, the Standard Daylight Coefficient model is prone to over- or under-estimation of illuminances, especially in instances where direct sunlight is incident on the measurement grid. This can be mostly attributed to the approximation of the sun position to 2305 locations (the DDS approach) through the Reinhart sky subdivision scheme as shown in Figure 5. The results from the improved method proposed by the authors were mostly in agreement with the benchmark results as precise positions of the sun were used in those cases. For the entire set of observations, two major outlying observations were recorded for Seattle, WA. As shown in Figure 17, the results from all the methods deviated by around -40% and higher for 13:30 on June 21st and 15:30 on September 22nd. These anomalies in observation can be attributed to low levels of illuminance prevailing on the workplane during those times. These values are visualized in Figure 18 and Figure 19.

The data plotted in Figure 12 to Figure 17 also indicates that there are no definitive performance trends associated with the use of skies with higher resolutions. The deviations in values relating to MF1, MF2 … MF6 in these figures correspond to skies with Reinhart subdivisions of 145, 577 … 3605 respectively. It can thus be argued that within the context of the employed sky subdivision scheme, no particular improvements in accuracy can be obtained through the use of high resolution skies. As shown in Figure 20, the use of higher sky resolutions, however, does entail a corresponding increase in the simulation runtime.

Figure 18. Illuminance values measured for Seattle on 21st June on 13:30Hrs.
Figure 19. Illuminance values measured for Seattle on 22nd September at 15:30 hrs.

Figure 20. Simulation runtime (for the set of 240 simulations summarized in Figure 12 to Figure 17) plotted as a function of sky patches used in the simulation. The runtimes plotted above were from simulations that were performed with a Linux operation system on an Intel i7 3770 3.4GHz Desktop computer with 8 virtual cores. The use of higher resolution skies was accompanied by a corresponding increase in the value of ambient divisions (-ad) used for the simulation. The runtime for the MF1 simulation was 272.3 seconds. The corresponding runtime for the simulation using the Standard Daylight Coefficient model was 245.2 seconds.
Based on the results discussed thus far, following inferences can be made:

1. When compared with the Standard Daylight Coefficient Model, the accuracy-related benefits of the improved method proposed by the authors are mostly related to the precision with which the solar discs are incorporated into the simulation.
2. Under the currently followed Reinhart sky subdivision scheme, there appear to be no tangible benefits of employing higher resolution skies for simulations.

Keeping the above points in consideration, a series of Annual Sunlight Exposure (ASE) simulations performed with the Standard Daylight Coefficient Model and an improved model consisting of 145 sky subdivisions and precise sun positions. The objective of these simulations was to identify any differences in an annual sunlight-based metric obtained through the improved model and the standard model. As shown by the values listed in Table 4, the improved model predicted values that were different from the standard model by 1 or 2% for Denver, Fargo, Phoenix and Pittsburgh. Although Birmingham and Phoenix are on similar latitudes, their ASE values differed by nearly 6%. This difference can be attributed to the low direct-normal radiation values in the dataset for Birmingham, especially in the morning hours. The values for direct-normal radiations for Phoenix and Birmingham are plotted in Figure 21.

Table 4. Annual Sunlight Exposures calculated through the Standard Daylight Coefficient Model (DDS Method) and the improved method proposed by the authors. MF1 relates to 145 sky patches.

<table>
<thead>
<tr>
<th>Location</th>
<th>DDS Method</th>
<th>Improved (MF1) Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Birmingham, AL</td>
<td>0.32</td>
<td>0.32</td>
</tr>
<tr>
<td>Denver, CO</td>
<td>0.47</td>
<td>0.49</td>
</tr>
<tr>
<td>Fargo, ND</td>
<td>0.46</td>
<td>0.47</td>
</tr>
<tr>
<td>Phoenix, AZ</td>
<td>0.38</td>
<td>0.4</td>
</tr>
<tr>
<td>Pittsburgh, PA</td>
<td>0.28</td>
<td>0.29</td>
</tr>
<tr>
<td>Seattle, WA</td>
<td>0.28</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Figure 21. Annual sun-path diagrams for Phoenix AZ and Birmingham AL. The colors indicate the direct normal radiation values. As indicated by the blue regions in the east direction for Birmingham’s dataset, when compared with Birmingham for the same period, considerably less direct normal radiation will be incident on a south facing space in the mornings.
6. Conclusion

The analysis from the previous sections indicate that the proposed improvements to the Daylight Coefficient Method can provide more accurate results for annual daylighting simulations. The wide deviations of up to 50% in illuminance values observed in the results from the Standard Daylight Coefficient Model can be eliminated by using precise sun positions at a marginal expense of increased runtime. The incorporation of additional direct suns in a simulation is associated with additional shadow testing calculations that are purely deterministic in nature (Ward 1994). Compared to the computing resources available in the early 1980s, modern computers can perform such calculations in a relatively short time. The average runtimes for the MF1 model simulations listed in Table 4 were approximately 67.7 seconds. These runtimes exceeded those for the direct simulation aspect of the DDS method by only 11.09%.

The qualifying criteria for obtaining daylighting-based credits in the LEEDv4 standard is affected by an increase or decrease of 1-2 % in ASE. So, it is imperative that, when available, precise methods like the one proposed through this research be used for such simulations.

The current research studied the impact of improved method on a simplistic south-facing model. Further investigations will be required to ascertain the benefits of such simulations to models involving complex surfaces and geometries, as well as windows that are oriented in all cardinal directions.

6. References

Ai Q. 2016. The Impact of Overhang Length, Window Orientation, and Climate on Spatial Daylight Autonomy (SDA) and Annual Sunlight Exposure (ASE) for a Classroom. The Pennsylvania State University.


Brembilla E, Mardaljevic J, Hopfe CJ. Sensitivity Analysis studying the impact of reflectance values assigned in Climate-Based Daylight Modelling. The 14th International Conference of the International Building Performance Simulation Association; 2015b.


IES. 2012. LM-83-12 IES Spatial Daylight Autonomy (sDA) and Annual Sunlight Exposure (ASE). New York, NY, USA.


Roudsari M, Subramaniam S. Automating radiance workflows using Python. 15th International Radiance Workshop; 2016; Padova.


Ward G. The Radiance rtcontrib Program [Internet]. LBNL; 2012.